

Name of the course : **CBCS B.Sc. (H) Mathematics**
 Unique Paper Code : **32357502**
 Name of Paper : **DSE-I: Mathematical modelling and graph theory**
 Semester : **V**
 Duration : **3 hours**
 Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

- Find the power series solution of the equation $(x^2 + 1)y'' + xy' - xy = 0$ in powers of x . Also discuss the singularities of the equation $x^2(x + 1)^2 y'' + (x^2 - 1)y' + 2y = 0$.
- Consider a spring-mass system governed by $my'' + \beta y' + ky = f(t)$, $y(0) = y'(0) = 0$. Suppose we apply a unit step force $f(t) = u(t)$ to the mass, initially at equilibrium and you observe the system respond as $y(t) = -\frac{1}{2}e^{-t} \cos t - \frac{1}{2}e^{-t} \sin t + \frac{1}{2}$. What are the physical parameters m , β and k ? Find $\mathcal{L}^{-1}\left(\frac{s^2+1}{s(s+1)^2}\right)$, $\mathcal{L}^{-1}\left(\frac{s}{s^2+9} + \frac{5}{s^2+9}\right)$. Is the function $-6te^{\frac{3t}{2}} \sin t$, $t \geq 0$ of exponential order? Justify your claim.
- Using Monte Carlo simulation, write an algorithm to calculate the area under the curve $y = 2x^2 = 2$, and above x -axis on the interval $0 \leq x \leq 4$.

Consider a small harbor with unloading facilities for ships, where only one ship can be unloaded at any time. The unloading time required for a ship depends on the type and the amount of cargo. Below is given a situation with 6 ships

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5	Ship 6
Time between successive ships	10	25	15	30	115	55
Unload time	15	50	25	70	60	80

Draw the timeline diagram depicting clearly the situation for each ship. Also determine length of longest queue, average waiting time for ships and total time in which docking facilities are idle.

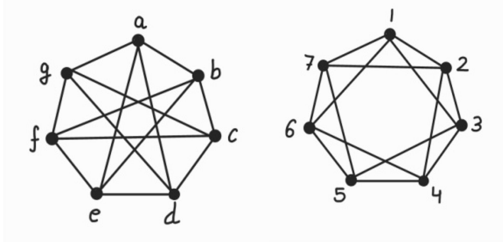
- Using simplex method, solve the following linear programming problem

$$\begin{aligned} &\text{Maximize} && 8x_1 + 12x_2 \\ &\text{subject to} && -10x_1 - 20x_2 \geq -140 \\ &&& 6x_1 + 8x_2 \leq 72 \\ &&& x_1, x_2 \geq 0. \end{aligned}$$

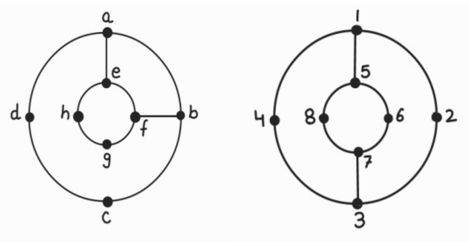
Determine the sensitivity of the optimal solution to a change c_1 using the objective function $c_1x_1 + 12x_2$.

5. Are the following pairs of graphs isomorphic? If so, find a suitable one-one correspondence between the vertices of graphs: if not, explain why no such correspondence exists. Determine which of the given four graphs are Eulerian and/or Hamiltonian. Write down the corresponding Eulerian trail and/or Hamiltonian cycle, where possible.

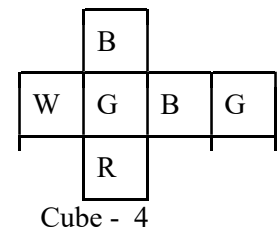
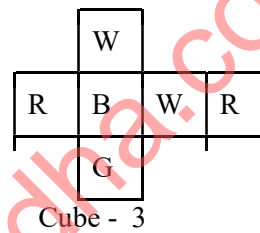
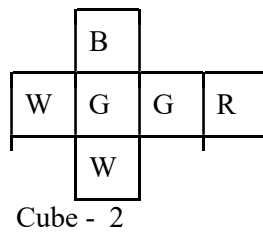
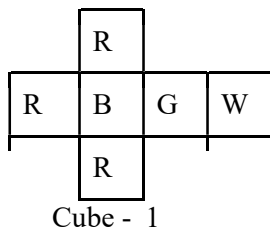
Pair: 1



Pair: 2



6. Find a solution to the four-cube problem for the following set of cubes:



Draw a graph representing predator behavior in the following problem: Snakes eat frogs and birds eat spiders; birds and spiders both eat insects; frogs eat snails, spiders and insects. Is this graph bipartite? Is this a regular graph? Justify your answers.